

Automatic code generation - developing high performance propagators better, faster and cheaper.

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Devito - Automated finite difference propagators

Something is rotten in the state of Denmark...

Seismic inversion is extremely computationally demanding!

Yet new models are built around bespoke operators...

- Discretization and numerical methods are chosen a priori ¹
- Performance optimization repeated for each architecture
- Requires many person-months (years) to develop new algorithms

Complex algorithms need end-to-end optimization

- Optimization at various levels of expertise
- Domain-specialists, numericists and compiler experts ...
- But we can't all be polymaths. We need separation of concerns!

¹M. Louboutin, M. Lange, F. J. Herrmann, N. Kukreja, and G. Gorman. Performance prediction of finite-difference solvers for different computer architectures. *Computers and Geosciences*, 105:148 – 157, 2017

Devito - Automated finite difference propagators

Symbolic computation is a powerful tool!

- FEniCS / Firedrake - Finite element DSL packages

Velocity-stress formulation of elastic wave equation, with isotropic stress:

$$\rho \frac{\partial \mathbf{u}}{\partial t} = \nabla \cdot \mathbb{T}$$

$$\frac{\partial \mathbb{T}}{\partial t} = \lambda (\nabla \cdot \mathbf{u}) \mathbb{I} + \mu (\nabla \mathbf{u} + \nabla \mathbf{u}^T)$$

Weak form of equations written in UFL¹:

```
F_u = density*inner(w, (u - u0)/dt)*dx - inner(w, div(s0))*dx
solve(lhs(F_u) == rhs(F_u), u)
```

¹ Anders Logg, Kent-Andre Mardal, and Garth Wells. *Automated Solution of Differential Equations by the Finite Element Method: The FEniCS Book*. Springer Publishing Company, Incorporated, 2012

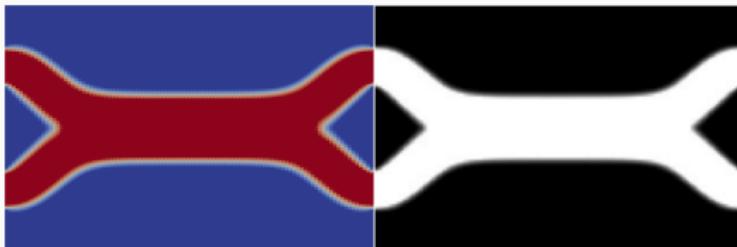
Devito - Automated finite difference propagators

Symbolic computation is a powerful tool!

Dolfin-Adjoint: Symbolic adjoints from symbolic PDEs¹

- Solves complex optimisation problems
- 2015 Wilkinson prize winner

Below is the optimal design of a double pipe that minimises the dissipated power in the fluid.



¹P. E. Farrell, D. A. Ham, S. W. Funke, and M. E. Rognes. Automated derivation of the adjoint of high-level transient finite element programs. *SIAM Journal on Scientific Computing*, 35(4):C369–C393, 2013

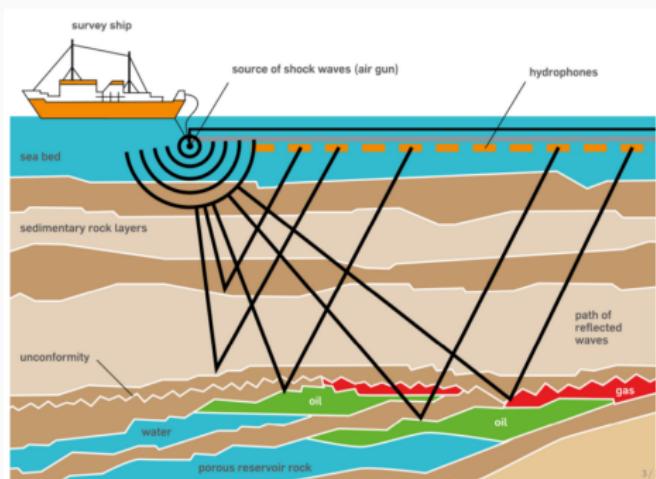
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For Seismic imaging we need to solve inversion problems

- Finite difference solvers for forward and adjoint runs
- Different types of wave equations with large complicated stencils

Many stencil languages exist, but few are practical

- Stencil still written by hand!



Devito - Automated finite difference propagators

- SymPy - Symbolic computer algebra system in pure Python¹
- Features:
 - Complex symbolic expressions as Python object trees
 - Symbolic manipulation routines and interfaces
 - Convert symbolic expressions to numeric functions
 - Python (NumPy) functions; C or Fortran kernels
 - For a great overview see [A. Meurer's talk at SciPy 2016](#)

For specialised domains generating C code is not enough!

- Compiler-level optimizimizaton to leverage performance
- Stencil optimization is a research field of its own

¹Aaron Meurer, Christopher P Smith, Mateusz Paprocki, Ondřej Čertík, Matthew Rocklin, AMiT Kumar, Sergiu Ivanov, Jason K Moore, Sartaj Singh, Thilina Rathnayake, et al. Sympy: Symbolic computing in python. Technical report, PeerJ Preprints, 2016

Devito - Automated finite difference propagators

Devito: a finite difference DSL for seismic imaging

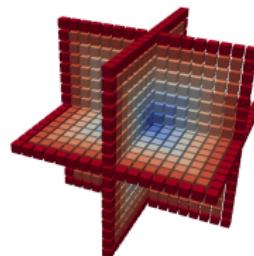
- Generates highly optimized stencil code
 - OpenMP threading and vectorisation pragmas
 - Cache blocking and auto-tuning
 - Symbolic stencil optimisation
- From concise mathematical syntax

Acoustic wave equation:

$$m \frac{\partial^2 u}{\partial t^2} + \eta \frac{\partial u}{\partial t} - \nabla u = 0$$

can be written as

```
eqn = m * u.dt2 + eta * u.dt - u.laplace
```



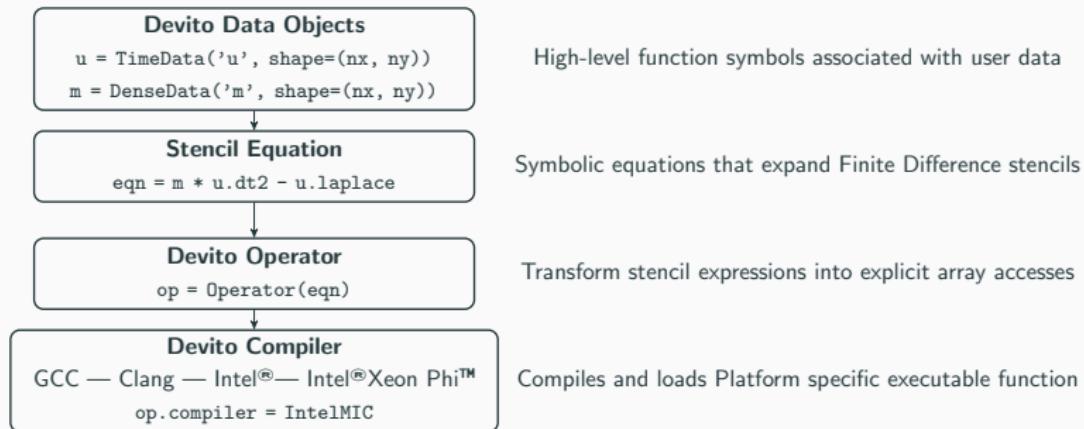
Devito - Automated finite difference propagators

Development is driven by real-world problems!

- Productivity through code generation
 - Variable numerical discretisation stencil size
 - Individual operators in 10s of lines of code
 - Complete problem setups in a few 100 lines
- Fast high-order operators for inversion problems
 - Automated performance optimisation
 - Customization through hierarchical API

Devito - Automated finite difference propagators

Development is driven by real-world problems!



Devito - Automated finite difference propagators

Wave propagators in less than 100 lines

```
def forward(model, m, eta, src, rec, order=2, save=True):
    # Create the wavefield function
    u = TimeData(name='u', shape=model.shape, save=save,
                  time_order=2, space_order=order)

    # Derive stencil from symbolic equation
    eqn = m * u.dt2 - u.laplace + eta * u.dt
    stencil = solve(eqn, u.forward)[0]
    update_u = [Eq(u.forward, stencil)]

    # Inject wave as source term
    src_term = src.inject(field=u, expr=src * dt**2 / m)

    # Interpolate wavefield onto receivers
    rec_term = rec.interpolate(expr=u)

    # Create operator with source and receiver terms
    return Operator(update_u + src_term + rec_term,
                   subs={s: dt, h: model.spacing})
```

Devito - Automated finite difference propagators

Wave propagators in less than 100 lines

```
def adjoint(model, m, eta, srca, rec, order=2):
    # Create the adjoint wavefield function
    v = TimeData(name='v', shape=model.shape,
                  time_order=2, space_order=order)

    # Derive stencil from symbolic equation
    eqn = m * v.dt2 - v.laplace - eta * v.dt
    stencil = solve(eqn, u.forward)[0]
    update_v = [Eq(v.backward, stencil)]

    # Inject the previous receiver readings
    rec_term = rec.inject(field=v, expr=rec * dt**2 / m)

    # Interpolate the adjoint-source
    srca_term = srca.interpolate(expr=v)

    # Create operator with source and receiver terms
    return Operator(update_v + rec_term + srca_term,
                   subs={s: dt, h: model.spacing},
                   time_axis=Backward)
```

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Wave propagators in less than 100 lines

```
def gradient(model, m, eta, srca, rec, order=2):
    # Create the adjoint wavefield function
    v = TimeData(name='v', shape=model.shape,
                  time_order=2, space_order=order)

    # Derive stencil from symbolic equation
    eqn = m * v.dt2 - v.laplace - eta * v.dt
    stencil = solve(eqn, u.forward)[0]
    update_v = [Eq(v.backward, stencil)]

    # Inject the previous receiver readings
    rec_term = rec.inject(field=v, expr=rec * dt**2 / m)

    # Gradient update terms
    grad = DenseData(name='grad', shape=model.shape)
    grad_update = Eq(grad, grad - u.dt2 * v)

    # Create operator with source and receiver terms
    return Operator(update_v + [grad_update] + rec_term
                   subs={s: dt, h: model.spacing},
                   time_axis=Backward)
```

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Reverse time migration in less than 100 lines

```
# Create the true and a smoothed model
m_true = Model(...)
m_smooth = Model(...)

# Create operators for forward and gradient
op_forward = forward(...)
op_gradient = forward(...)

# Create gradient field and loop over shots
grad = DenseData(name='grad', shape=model.shape)

for shot in shots:
    # Create receiver data from true model
    src = PointData(shot.source, ...)
    rec_true = PointData(shot.receiver.coordinates, ...)
    op_forward(src=src, rec=rec_true, m=m_true)

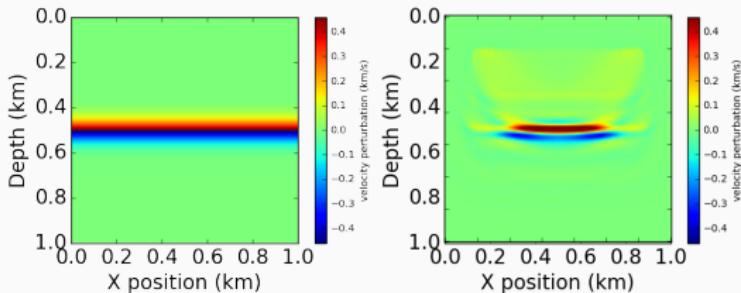
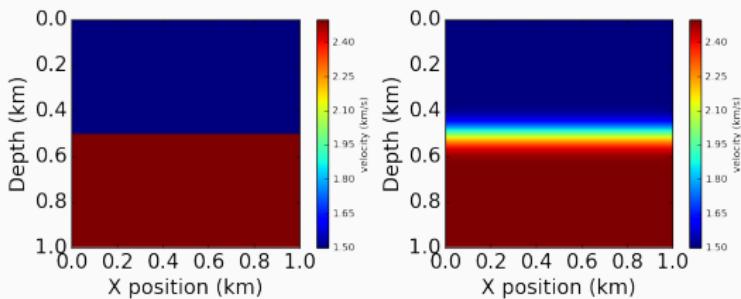
    # Run forward modelling operator with smooth model
    u = TimeData(name='u', shape=model.shape,
                 time_order=2, space_order=order)
    rec_smooth = PointData(shot.receiver.coordinates, ...)
    op_forward(u=u, src=src, rec=rec_smooth, m=m_smooth)

    # Compute gradient update from the residual
    v = TimeData(name='v', shape=model.shape,
                 time_order=2, space_order=order)
    residual = rec_true.data[:] - rec_smooth.data[:]
    op_gradient(u=u, v=v, grad=grad, rec=residual, m=m_smooth)
```

Devito - Automated finite difference propagators

Rapid propagator development and integration

- Test and verify in Python
- Operators in < 20 lines
- RTM loop in < 100 lines
- Variable stencil order



Devito - Automated finite difference propagators

From math to tuned HPC code in a few lines:

$$\frac{m}{\rho} \frac{d^2 p(x, t)}{dt^2} - (1 + 2\epsilon)(G_{\bar{x}\bar{x}} + G_{\bar{y}\bar{y}})p(x, t) - \sqrt{(1 + 2\delta)}G_{\bar{z}\bar{z}}r(x, t) = q,$$

$$\frac{m}{\rho} \frac{d^2 r(x, t)}{dt^2} - \sqrt{(1 + 2\delta)}(G_{\bar{x}\bar{x}} + G_{\bar{y}\bar{y}})p(x, t) - G_{\bar{z}\bar{z}}r(x, t) = q,$$

$$p(., 0) = 0,$$

$$\frac{dp(x, t)}{dt}|_{t=0} = 0,$$

$$r(., 0) = 0,$$

$$\frac{dr(x, t)}{dt}|_{t=0} = 0,$$

(incomplete) specification of a
TTI (Tilted Transverse Isotropy)
forward operator

$$D_{x1} = \cos(\theta)\cos(\phi)\frac{d}{dx} \Big|_l + \cos(\theta)\sin(\phi)\frac{d}{dy} \Big|_l - \sin(\theta)\frac{d}{dz} \Big|_l$$

$$D_{x2} = \cos(\theta)\cos(\phi)\frac{d}{dx} \Big|_l + \cos(\theta)\sin(\phi)\frac{d}{dy} \Big|_l - \sin(\theta)\frac{d}{dz} \Big|_l$$

$$G_{\bar{x}\bar{x}} = \frac{1}{2} \left(D_{x1}^T \left(\frac{1}{\rho} \right) D_{x1} + D_{x2}^T \left(\frac{1}{\rho} \right) D_{x2} \right)$$

rotated second order
differential operators

1

¹Y. Zhang and F. Mueller. Auto-generation and auto-tuning of 3d stencil codes on gpu clusters. In *Proceedings of the Tenth International Symposium on Code Generation and Optimization*, CGO '12, pages 155–164, New York, NY, USA, 2012. ACM

Devito - Automated finite difference propagators

From math to tuned HPC code in a few lines:

```
ang0, ang1 = cos(theta), sin(theta)
ang2, ang3 = cos(phi), sin(phi)
Gyp = (ang3 * u.dx - ang2 * u.dyr)
Gyy = (first_derivative(Gyp * ang3, dim=x, side=centered, order=space_order, matvec=transpose) -
       first_derivative(Gyp * ang2, dim=y, side=right, order=space_order, matvec=transpose))
Gyp2 = (ang3 * u.dxr - ang2 * u.dy)
Gyy2 = (first_derivative(Gyp2 * ang3, dim=x, side=right, order=space_order, matvec=transpose) -
       first_derivative(Gyp2 * ang2, dim=y, side=centered, order=space_order, matvec=transpose))
Gxp = (ang0 * ang2 * u.dx + ang0 * ang3 * u.dyr - ang1 * u.dzr)
Gzr = (ang1 * ang2 * v.dx + ang1 * ang3 * v.dyr + ang0 * v.dzr)
Gxx = (first_derivative(Gxp * ang0 * ang2, dim=x, side=centered, order=space_order, matvec=transpose) +
       first_derivative(Gxp * ang0 * ang3, dim=y, side=right, order=space_order, matvec=transpose) -
       first_derivative(Gxp * ang1, dim=z, side=right, order=space_order, matvec=transpose))
Gzz = (first_derivative(Gzr * ang1 * ang2, dim=x, side=centered, order=space_order, matvec=transpose) +
       first_derivative(Gzr * ang1 * ang3, dim=y, side=right, order=space_order, matvec=transpose) +
       first_derivative(Gzr * ang0, dim=z, side=right, order=space_order, matvec=transpose))
Gxp2 = (ang0 * ang2 * u.dxr + ang0 * ang3 * u.dy - ang1 * u.dz)
Gzr2 = (ang1*ang2*v.dxr+ang1*ang3*v.dy+ang0*v.dz) dim=x, side=right, order=space_order, matvec=transpose) +
       first_derivative(Gxp2 * ang0 * ang3, dim=y, side=centered, order=space_order, matvec=transpose) -
       first_derivative(Gxp2 * ang1, dim=z, side=centered, order=space_order, matvec=transpose))
Gzz2 = (first_derivative(Gzr2 * ang1 * ang2, dim=x, side=right, order=space_order, matvec=transpose) +
       first_derivative(Gzr2 * ang1 * ang3, dim=y, side=centered, order=space_order, matvec=transpose) +
       first_derivative(Gzr2 * ang0, dim=z, side=centered, order=space_order, matvec=transpose))

Hp = -(.5*Gxx + .5*Gxx2 + .5 * Gyy + .5*Gyy2)
Hrz = -(.5*Gzz + .5 * Gzz2)
stencilp = 1.0 / (2.0 * m + s * damp) * (4.0 * m * u + (s * damp - 2.0 * m) * u.backward
    + 2.0 * s**2 * (epsilon * Hp + delta * Hrz))
stencirl = 1.0 / (2.0 * m + s * damp) * (4.0 * m * v + (s * damp - 2.0 * m) * v.backward
    + 2.0 * s**2 * (delta * Hp + Hrz))
```

Devito - Automated finite difference propagators

From math to tuned HPC code in a few lines:

```
def forward(model, m, eta, epsilon, delta, theta, phi, src, rec, order=2):

    # Create two wavefields
    u = TimeData(name='u', shape=model.shape, time_order=2, space_order=order)
    v = TimeData(name='v', shape=model.shape, time_order=2, space_order=order)

    # Create update expressions from stencil
    stencilp, stencilr = ...
    update_u = Eq(u.forward, stencilp)
    update_v = Eq(v.forward, stencilr)

    # Inject wave as source term
    src_term = src.inject(field=u, expr=src * dt**2 / m)
    src_term += src.inject(field=v, expr=src * dt**2 / m)

    # Interpolate wavefield onto receivers
    rec_term = rec.interpolate(expr=u)

    # Create operator with source and receiver terms
    return Operator([update_u, update_v] + src_term + rec_term,
                   subs={s: dt, h: model.spacing})
```

Devito - Automated finite difference propagators

Summary:

- Productivity through code generation
 - Acoustic operators in < 20 lines
 - TTI operators in < 100 lines
 - Variable discretization and stencil order
 - Fully executable Python code, easy to experiment
 - Complete problem setups in < 1000 lines
- Fast wave propagators for inversion problems
 - Highly efficient development through automation
 - Interoperability: Generated code is low-level C
 - **Automated performance optimisation**

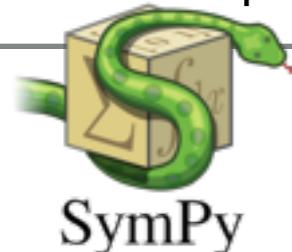


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Engineering and Physical Sciences
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The compilation flow: from symbolics to HPC code

Symbolic equations



Data objects



Analysis

DSE - Devito Symbolic Engine

Loop scheduler

DLE - Devito Loop Engine

Declarations, headers, ...

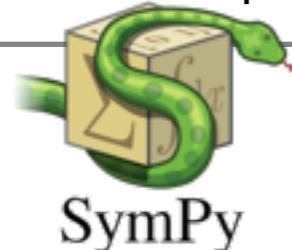
Code generation



C, MPI, OpenMP

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**“FLOPS”
OPTIMIZATIONS**

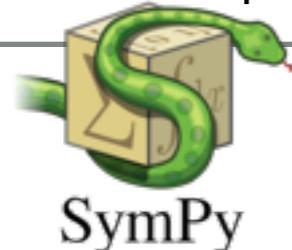


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C, MPI, OpenMP

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Code generation

**“FLOPS”
OPTIMIZATIONS**

**“MEMORY”
OPTIMIZATIONS**

C, MPI, OpenMP

Devito Symbolic Engine

A sequence of compiler passes to reduce FLOPS (no loops at this stage!)

Devito Symbolic Engine

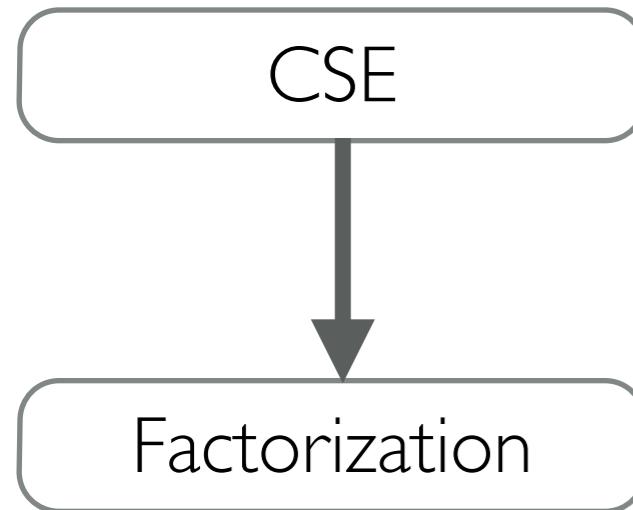
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CSE

- Common sub-expressions elimination
 - C compilers do it already... but necessary for symbolic processing and compilation speed

Devito Symbolic Engine

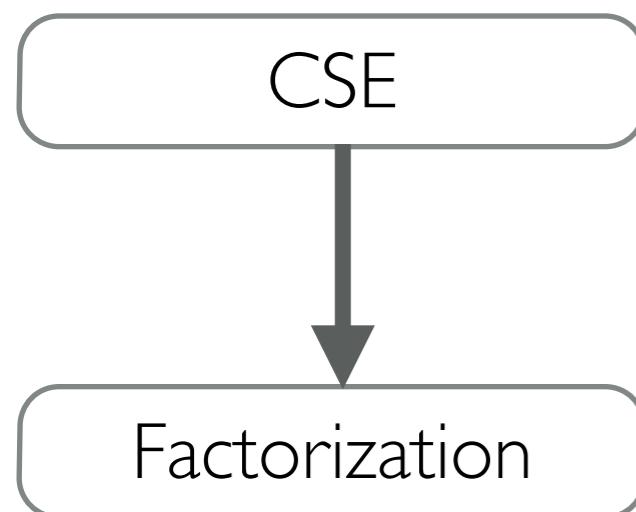
A sequence of compiler passes to reduce FLOPS (no loops at this stage!)



- Common sub-expressions elimination
 - C compilers do it already... but necessary for symbolic processing and compilation speed
- Heuristic factorization of recurrent terms
 - E.g., finite difference weights: $0.3*a + \dots + 0.3*b \Rightarrow 0.3*(a+b)$
 - Many possibilities (doesn't leverage domain properties yet!)

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Factorization impact:

TTI, space order 4: 1100 → 950

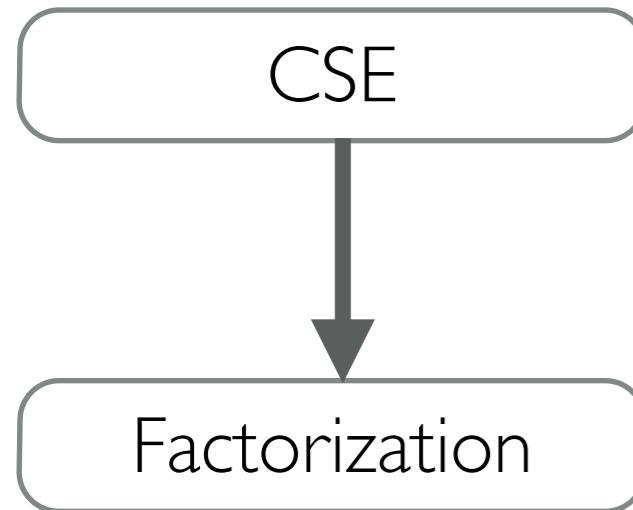
TTI, space order 8: 2380 → 2120

TTI, space order 12: 4240 → 3760

TTI, space order 16: 6680 → 5760

Devito Symbolic Engine

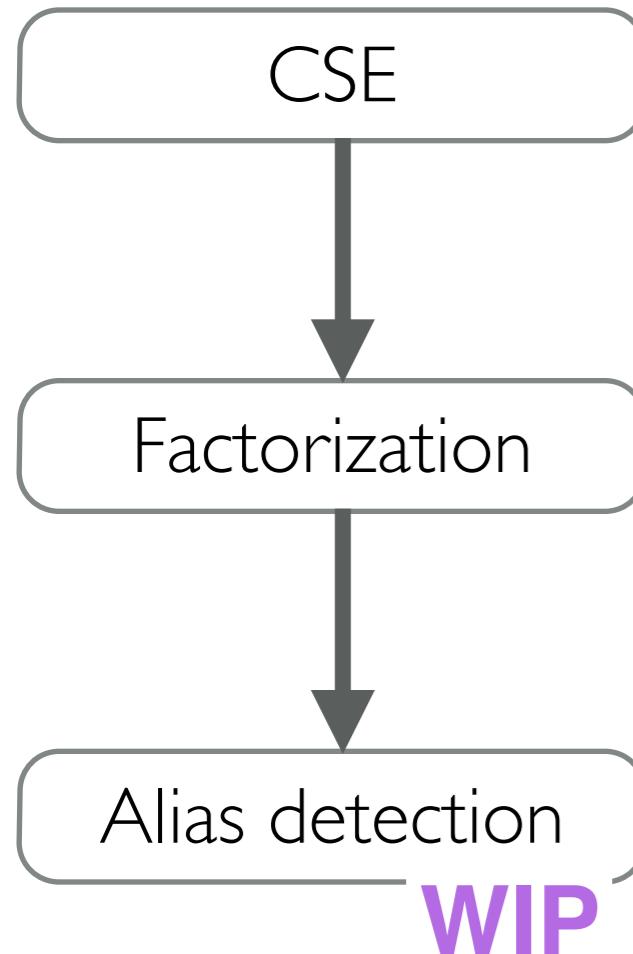
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 - Many possibilities (doesn't leverage domain properties yet!)
- Fundamental in compute-bound stencil codes (e.g., TTI)
 - E.g., `sin(phi[i,j,k])`, `sin(phi[i-1,j-1,k-1])`

DSE's aliases detection algorithms



```
tmp1 = ...*sin(phii,j,k) + ... + 0.4*sin(phii-1,j-1,k-1) + ... +  
...0.1*sin(phii+2,j+2,k+2) + ...
```

Observations (focus on underlined sub-expressions)

- Same operators (`sin`)
- Same operands (`phi`)
- Same indices (`i, j, k`)
- Linearly dependent index vectors (`[i, j, k], [i-1, j-1, k-1], [i+2, j+2, k+2]`)

DSE's aliases detection algorithms



Alias detection

Fundamental in compute-bound stencil codes (e.g., TTI)

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tmp1 = ...*sin(phii,j,k) + ... + 0.4*sin(phii-1,j-1,k-1) + ... +  
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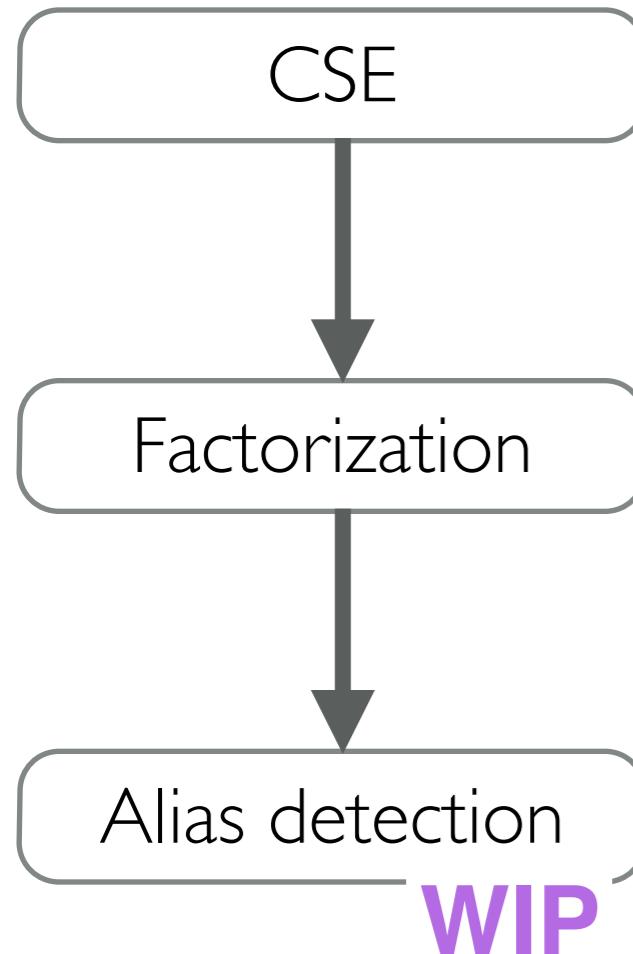
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- Same indices (`i, j, k`)
- Linearly dependent index vectors (`[i, j, k], [i-1, j-1, k-1], [i+2, j+2, k+2]`)

```
B[i,j,k] = sin(phi[i,j,k])
```

```
tmp1 = ...*B[i,j,k] + ... + 0.4*B[i-1,j-1,k-1] + ... + ... + 0.1*B[i+2,j+2,k+2] + ...
```

Devito Symbolic Engine

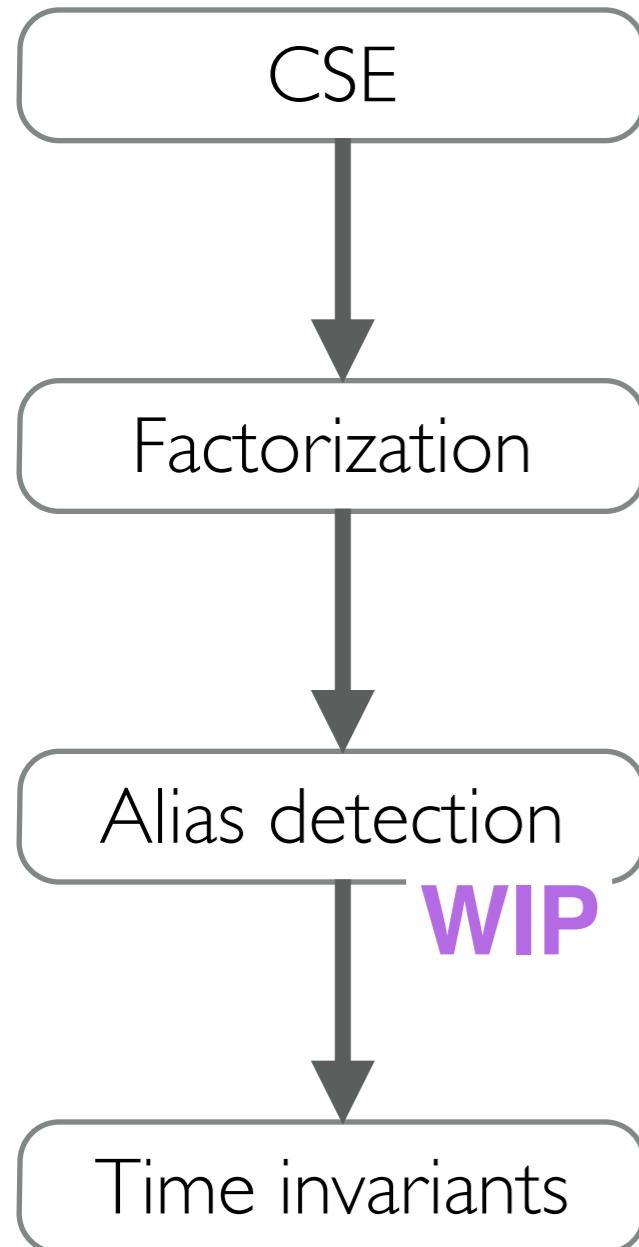
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- Fundamental in compute-bound stencil codes (e.g., TTI)
 - E.g., `sin(phi[i,j,k])`, `sin(phi[i-1,j-1,k-1])`
- Heuristic hoisting of time-invariant quantities
 - Currently, only (expensive) trigonometric functions applied to space-varying quantities

Devito Loop Engine

A sequence of compiler passes to introduce parallelism, SIMD vectorization and to improve data locality

Devito Loop Engine

A sequence of compiler passes to introduce parallelism, SIMD vectorization and to improve data locality

Cache opts

- Cache optimizations (mostly L1 cache)
 - Loop fission + elemental functions (register locality)
 - Padding + data alignment (split loads)

Devito Loop Engine

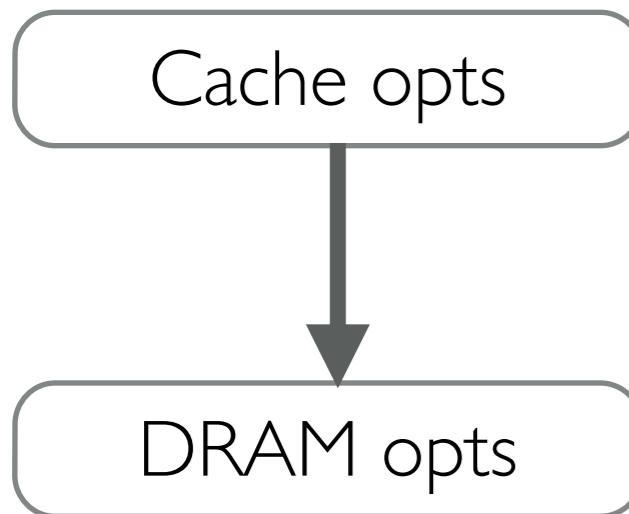
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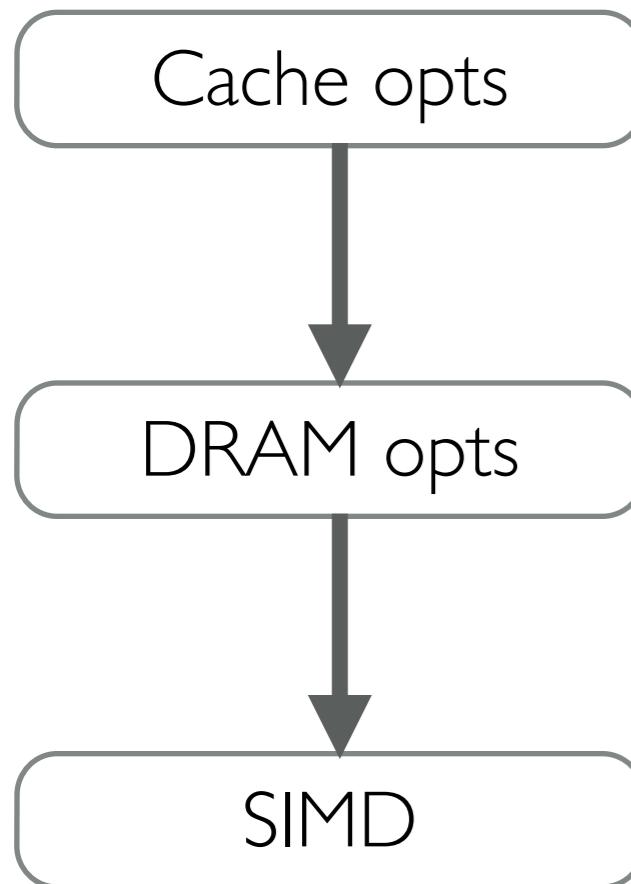
A sequence of compiler passes to introduce parallelism, SIMD vectorization and to improve data locality



- Cache optimizations (mostly L1 cache)
 - Loop fission + elemental functions (register locality)
 - Padding + data alignment (split loads)
- DRAM optimizations: loop blocking
 - 1D, 2D, 3D supported (but no time loop)
 - Auto-tuning supported

Devito Loop Engine

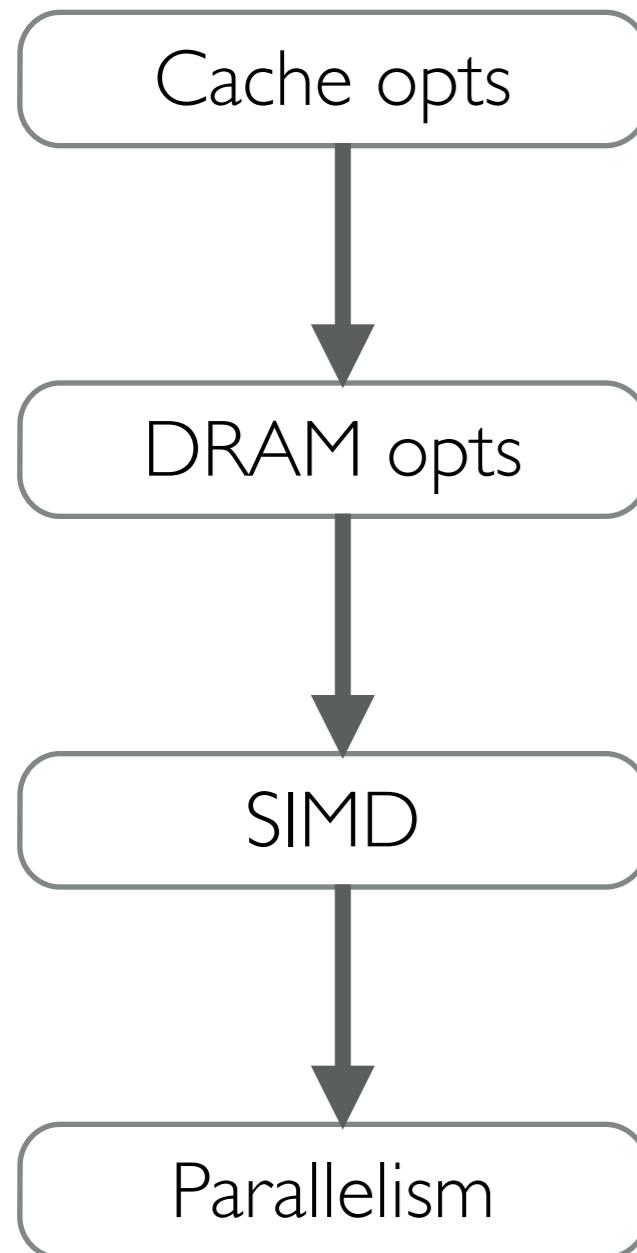
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- SIMD vectorization
 - Through compiler auto-vectorization
 - Why should I bother using intrinsics?
 - Various #pragmas introduced (e.g., ivdep, alignment, ...)

Devito Loop Engine

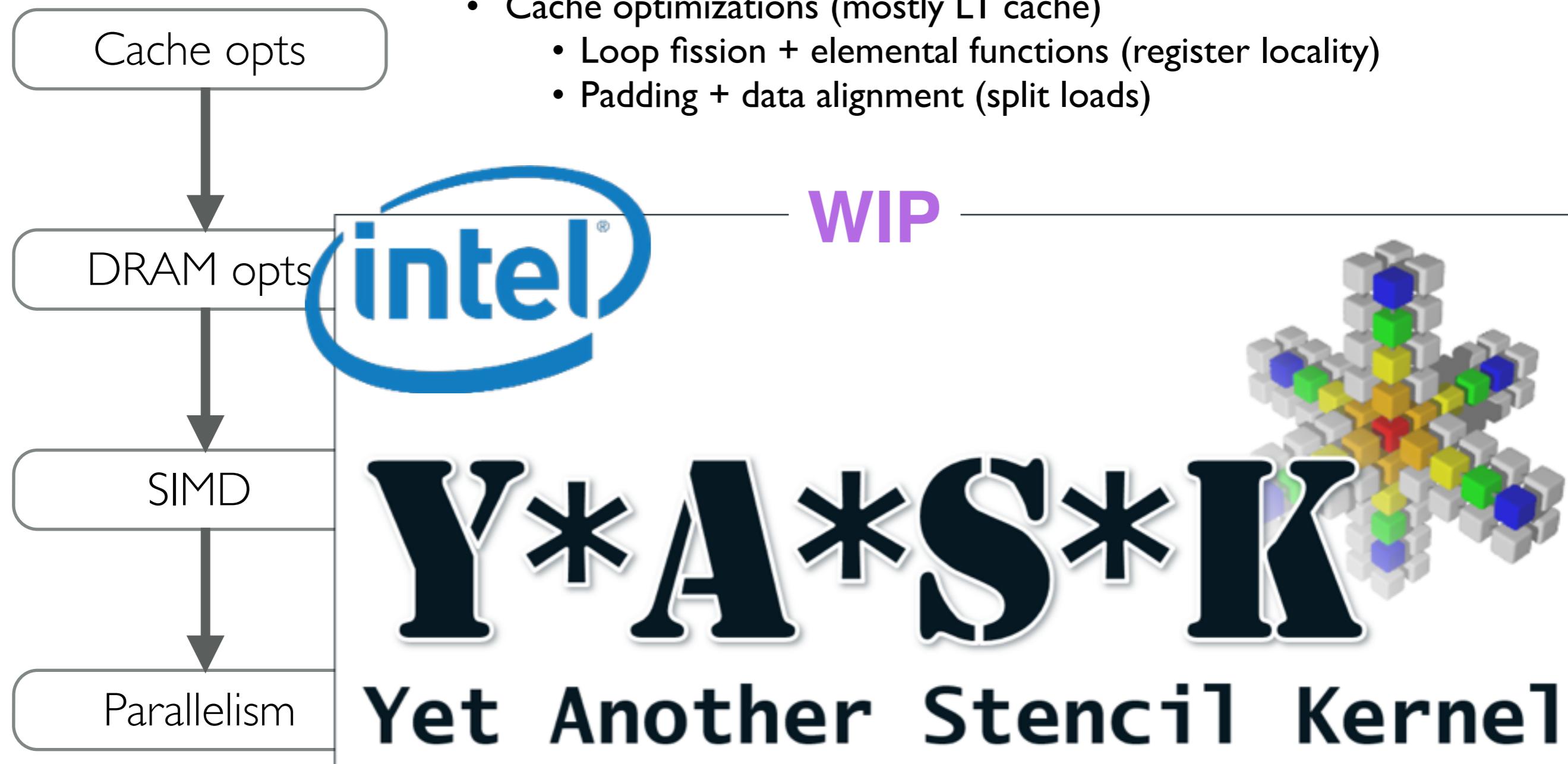
A sequence of compiler passes to introduce parallelism, SIMD vectorization and to improve data locality



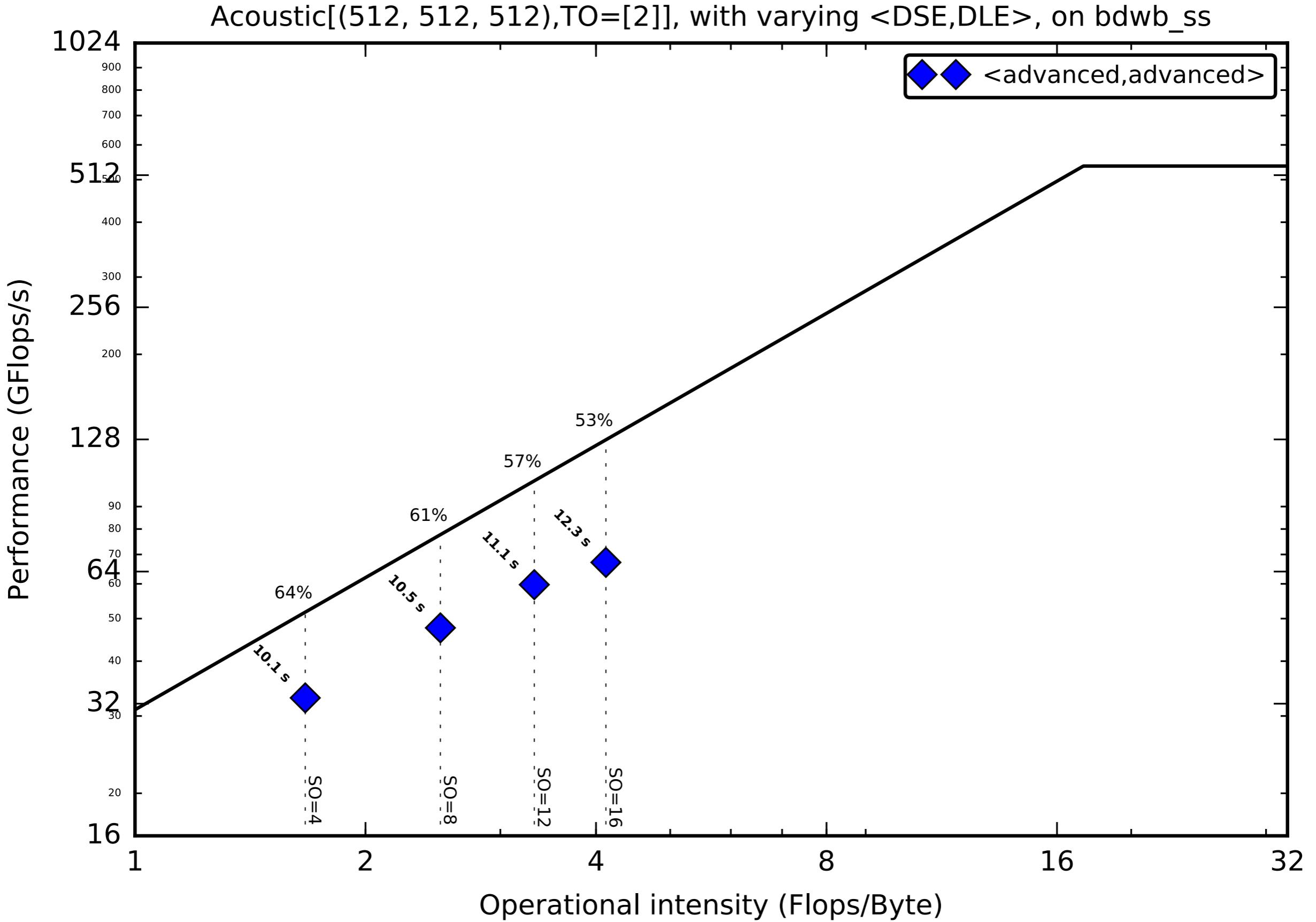
- Cache optimizations (mostly L1 cache)
 - Loop fission + elemental functions (register locality)
 - Padding + data alignment (split loads)
- DRAM optimizations: loop blocking
 - 1D, 2D, 3D supported (but no time loop)
 - Auto-tuning supported
- SIMD vectorization
 - Through compiler auto-vectorization
 - Why should I bother using intrinsics?
 - Various #pragmas introduced (e.g., ivdep, alignment, ...)
- OpenMP
 - #pragma collapse clause on the Xeon Phi

Devito Loop Engine

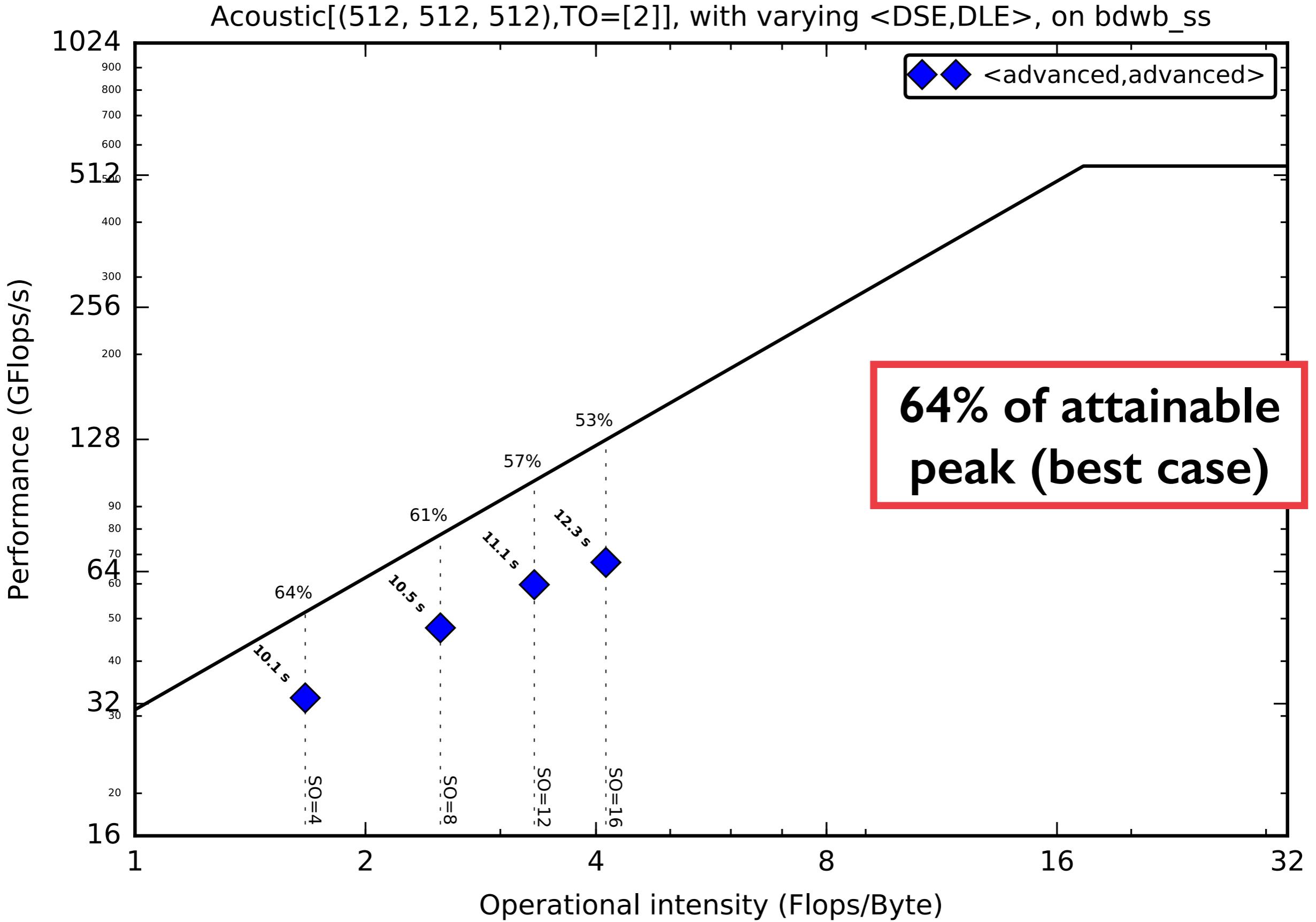
A sequence of compiler passes to introduce parallelism, SIMD vectorization and to improve data locality



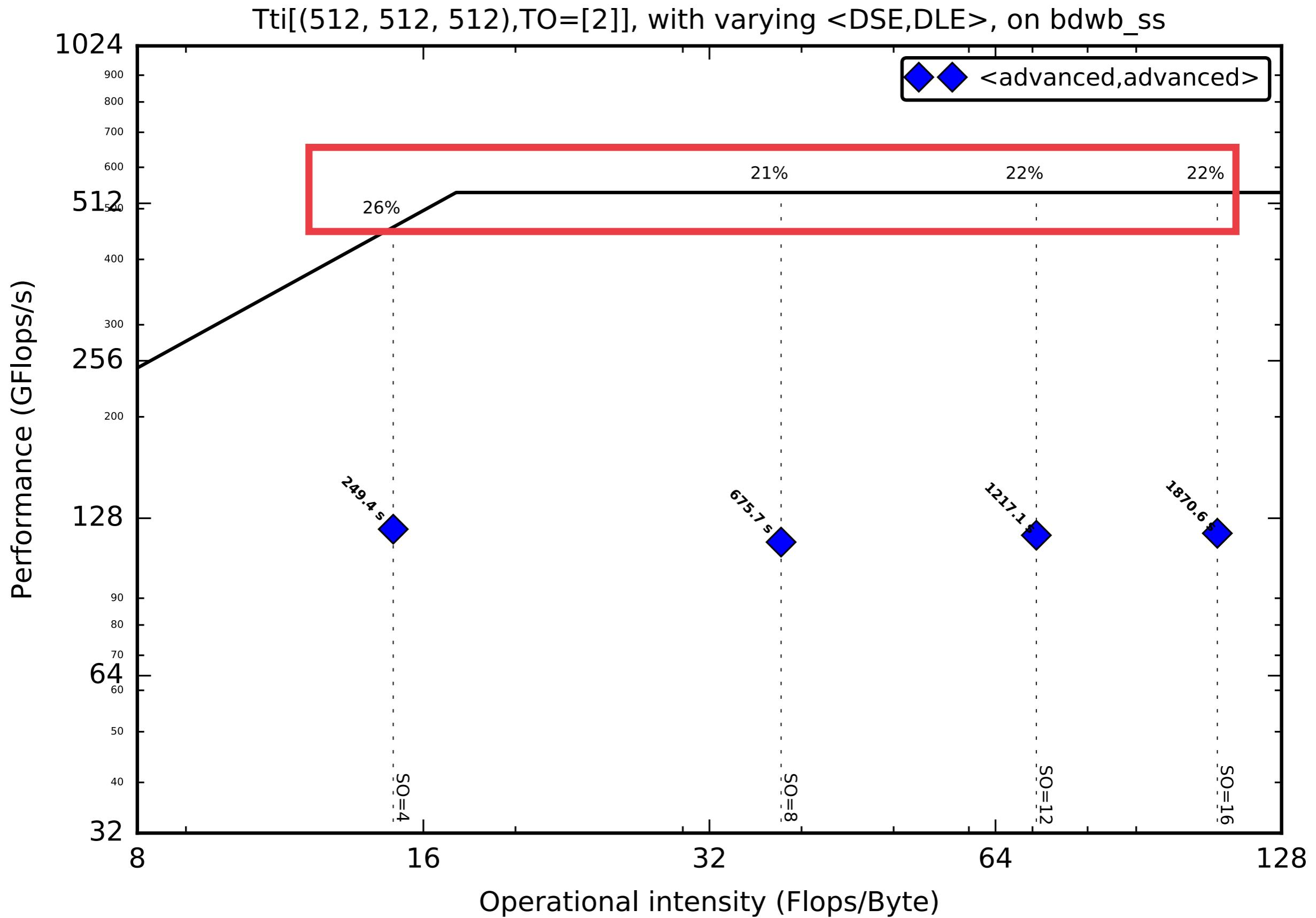
Acoustic on Broadwell



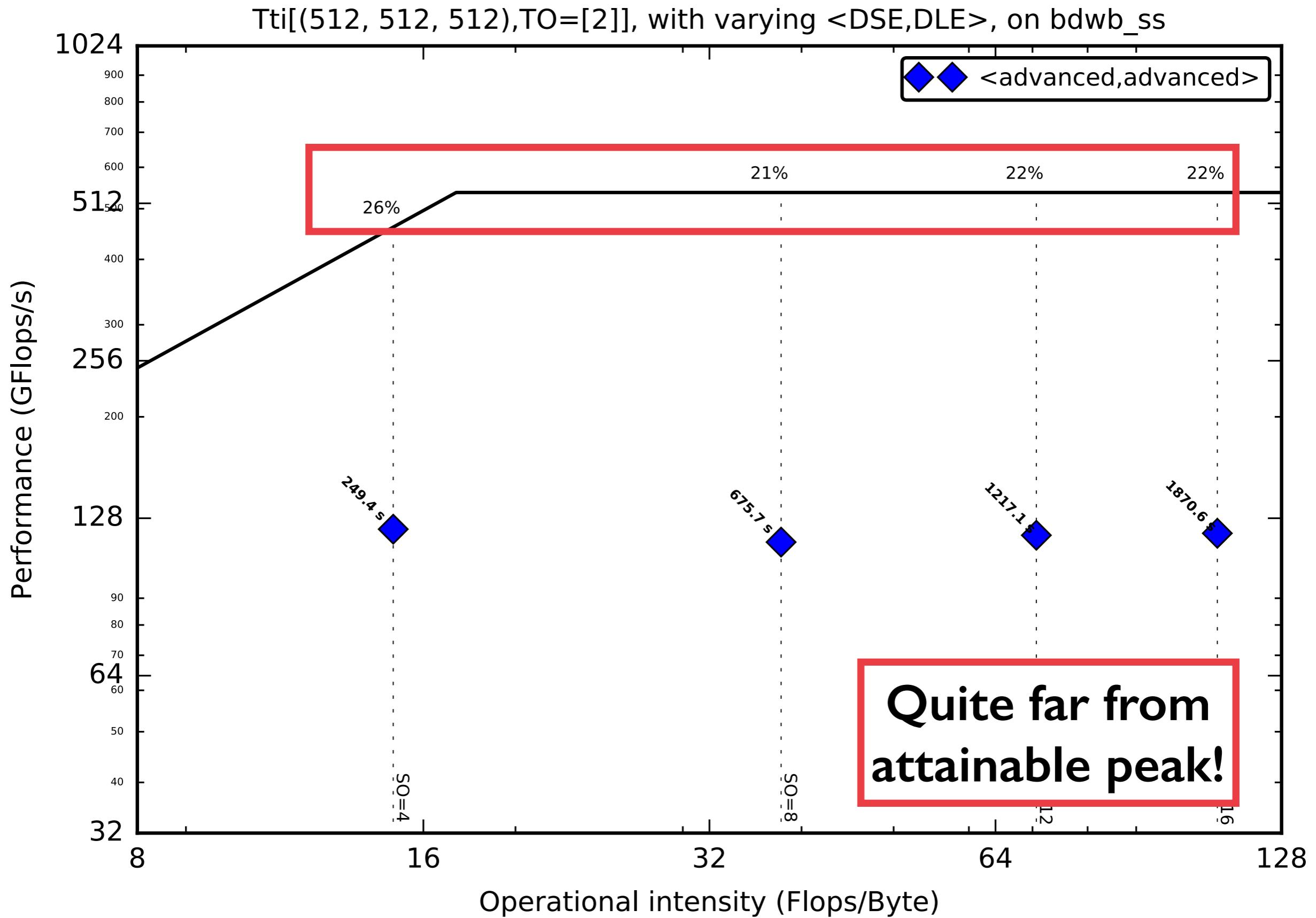
Acoustic on Broadwell



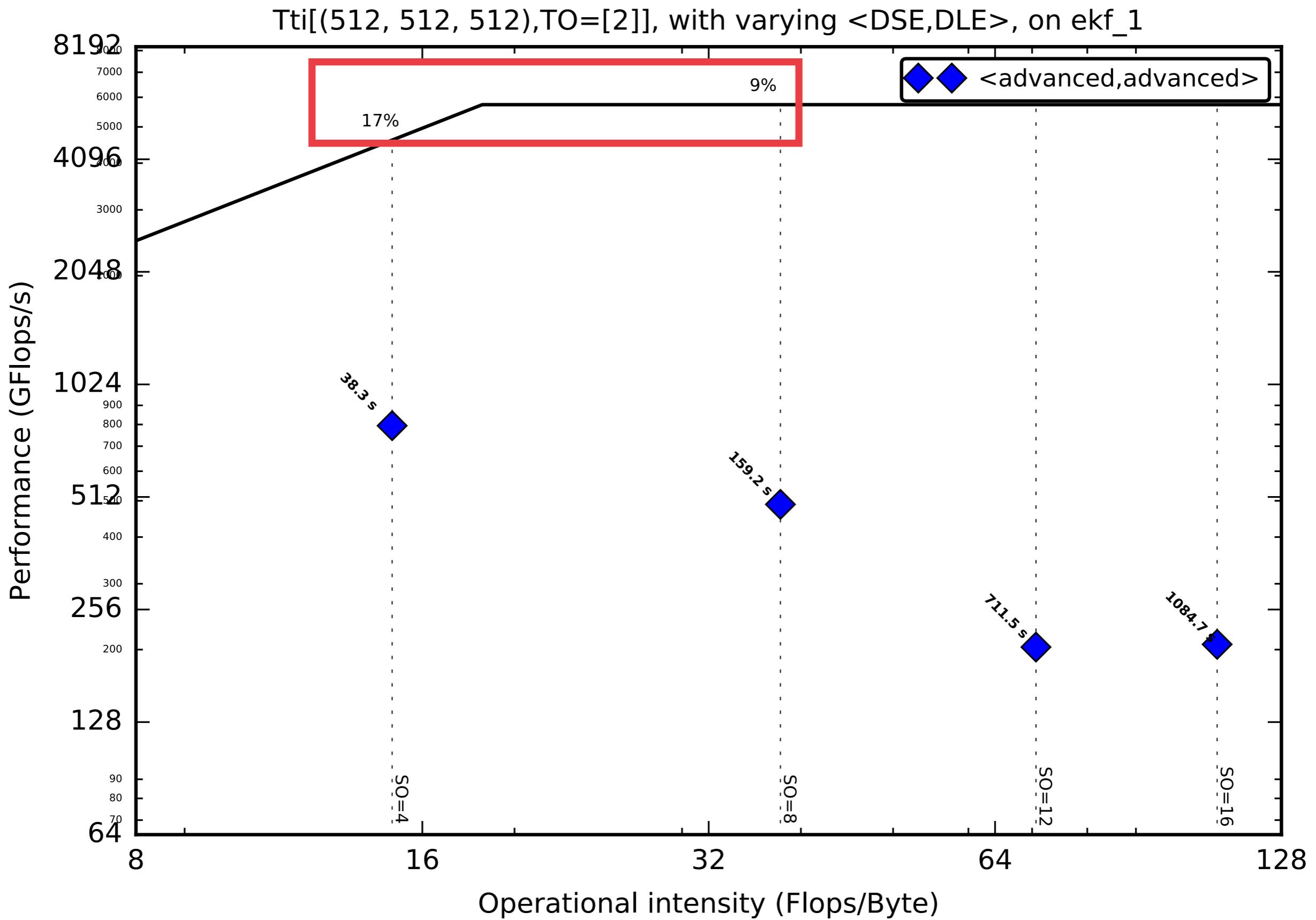
TTI on Broadwell (8 threads, single socket)



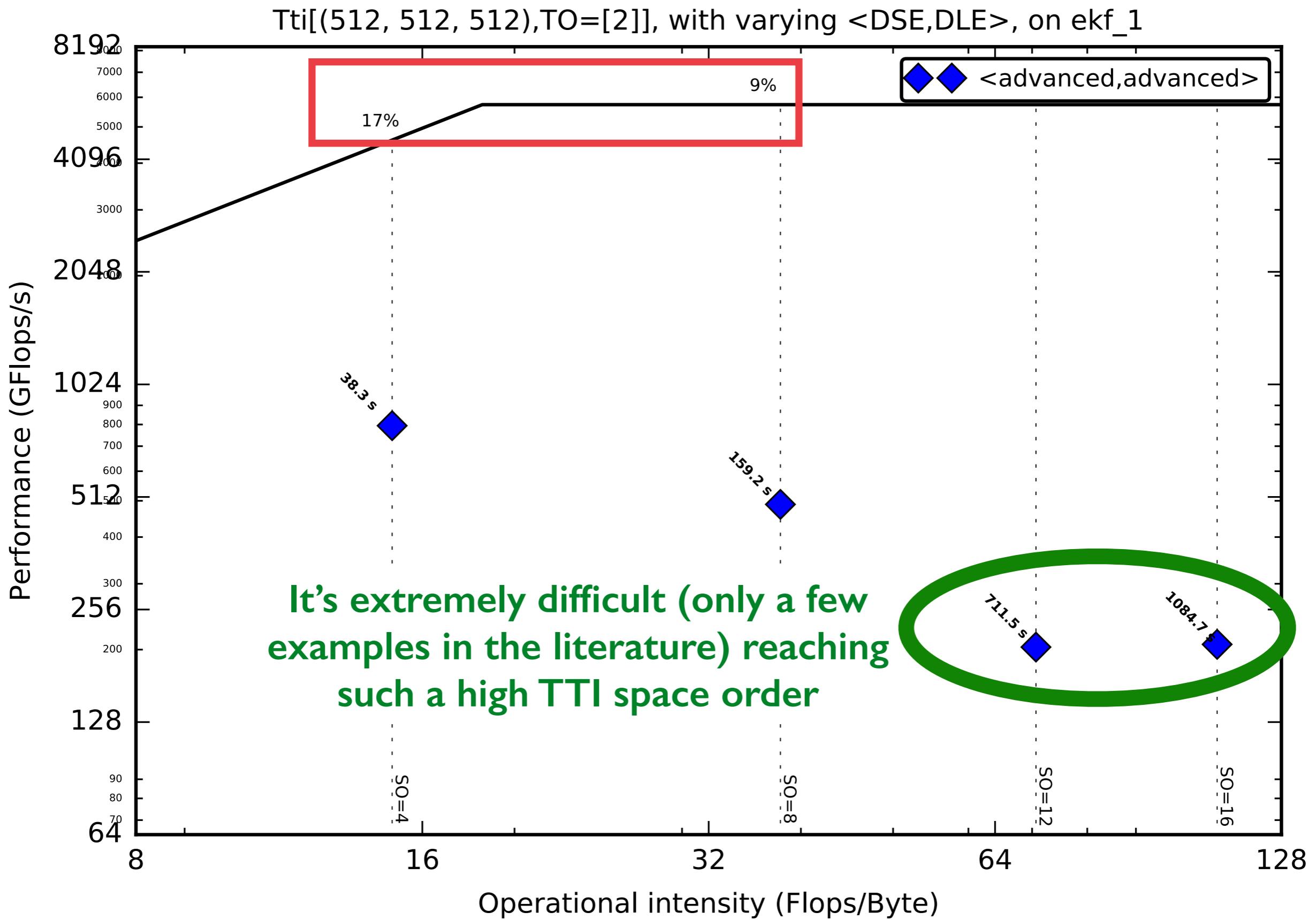
TTI on Broadwell (8 threads, single socket)



TTI on Xeon Phi (64 threads, cache mode, quadrant)



TTI on Xeon Phi (64 threads, cache mode, quadrant)



Conclusions and resources

- Devito: an efficient and sustainable finite difference DSL
- Driven/inspired by **real-world seismic imaging**
- **Interdisciplinary research effort**
- Based on **actual compiler technology**

Useful links

- <http://www.opesci.org>
- <https://github.com/opesci/devito>

